CHAPTER 26: CURRENT AND RESISTANCE

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Current and Resistance

What we will learn

- What is electric current (i)?
- What is current density (**J**) and drift speed (v_d)?
- What is resistance (R) and resistivity (ρ)?
- Ohm's law (V=i R)
- How to find the power in an electric circuit?

Current and Resistnace

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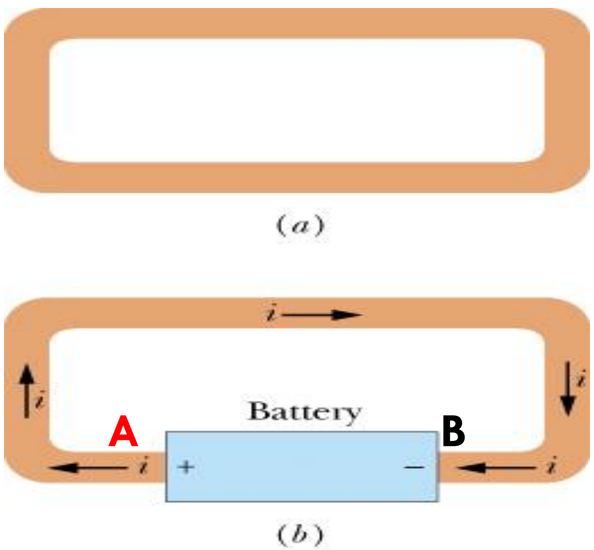
Lightning strikes the ground less than 1 km from a Virginia Tech football game. The chance of lightning striking a person directly is slim. The much greater danger lies in the ground current—the current that spreads out from the strike point. Everyone on the field or in the stands could have been knocked down, paralyzed, or killed by the ground current. If you are caught in the open during a lightning storm like this, there is a simple procedure for reducing your risk from ground current.

How can you reduce your risk from ground current?

The answer is in this chapter.

Electric Current



A *current* is a measure of amount of charge that moves past a point per unit time.

$$i \equiv \frac{dq}{dt} =$$

$$\left[\frac{\text{C}}{\text{s}}\right] \equiv [\text{Ampere}] = [\text{A}]$$

- In (a) all points are at the same potential.
- Free electrons inside the conductor move in random directions.
- No net charge transport.
- When inserting a battery, there will be a potential difference.
- The situation is no longer static. There is a net charge flow in a particular direction ← electric current!

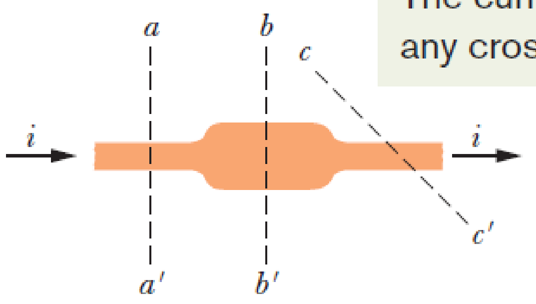
1 ampere = 1 A = 1 coulomb per second = 1 C/s.

Electric Current

$$i = \frac{dq}{dt} \qquad \text{(definition of current).}$$

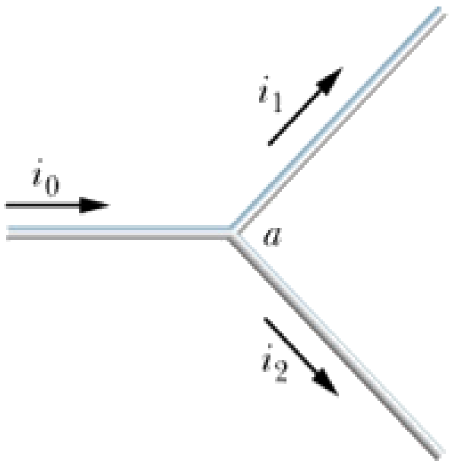
The charge that passes through the plane in a time interval extending from 0 to *t* is:

$$q = \int dq = \int_0^t i\, dt$$



- The current is the same for planes *aa'*, *bb'*, and *cc'* and for all planes that pass completely through the conductor, no matter what their location or orientation.
- Since charge is conserved, any electron passes through *aa' should pass through bb'*, and *cc'*.

Electric Current



The current into the junction must equal the current out (charge is conserved).

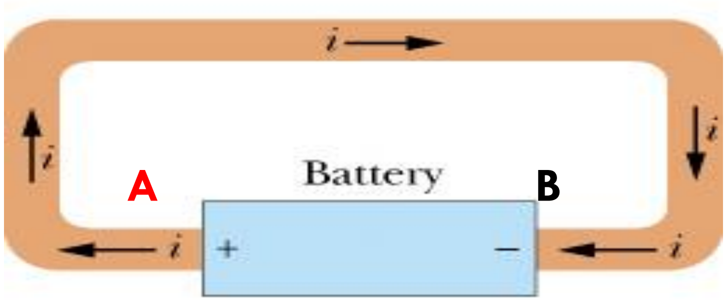
$$i_o = i_1 + i_2$$

This equation expresses the conservation of charge at point *a*. Note that we have not used vector addition.

Electric Current

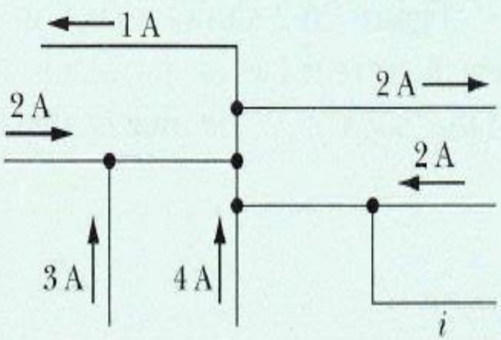
The direction of Current

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

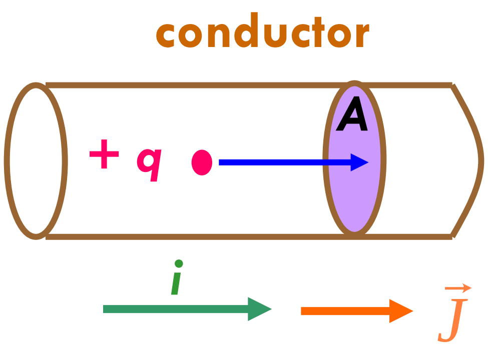


Electric Current

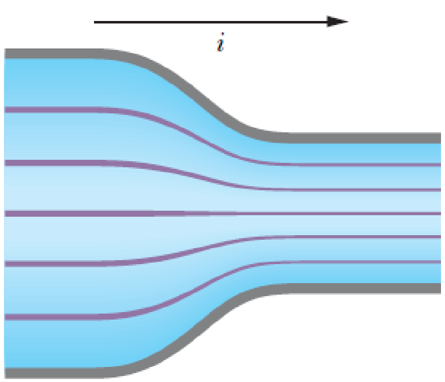
CHECKPOINT 1 The figure here shows a portion of a circuit. What are the magnitude and direction of the current *i* in the lower right-hand wire?



Current Density



J= the amount of current flowing through a cross-sectional area.



Current Density

Current density is a vector that is defined as follows:

Its magnitude is $J = \dfrac{i}{A} = \dfrac{current}{area}$

SI unit: $\mathrm{J} = \dfrac{\mathrm{A}}{\mathrm{m}^2}$

The direction of $\vec{J}$ is the same as that of the current.

The current through a conductor of cross-sectional area A is given by the equation $i = JA$ if the current density is constant.

$$i = \int \vec{J} \cdot d\vec{A}.$$

$$i = \int J\, dA = J \int dA = JA$$

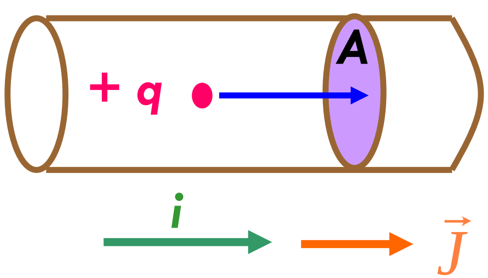
$$J = \frac{i}{A},$$

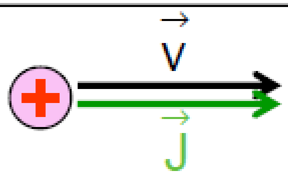
Current Density

Current density is a vector quantity

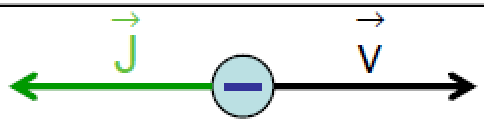
Direction

conductor





The same direction as the velocity of the moving positive charges.



Opposite to the direction of the velocity of the moving negative charges.

Drift Speed

If we assume that these charge carriers all move with the same drift speed v_d and that the current density $\boldsymbol{J}$ is uniform across the wire's cross-sectional area A, then the number of charge carriers in a length L of the wire is nAL. Here n is the number of carriers per unit volume.

The total charge of the carriers in the length L, each with charge e, is then

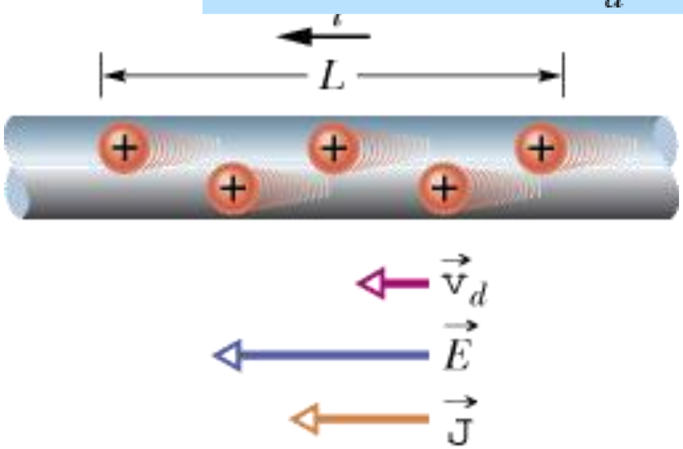
$$q = (nAL)e.$$

The total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

$$\Rightarrow \quad i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d.$$

$$v_d = \frac{i}{nAe} = \frac{J}{ne} \quad \Rightarrow \quad \vec{J} = (ne)\vec{v}_d.$$



When a current flows through a conductor the electric field causes the charges to move with a constant drift speed v_d.

Current Density

CHECKPOINT 2 The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current i, (b) the current density $\vec{J}$, (c) the electric field $\vec{E}$ in the wire?

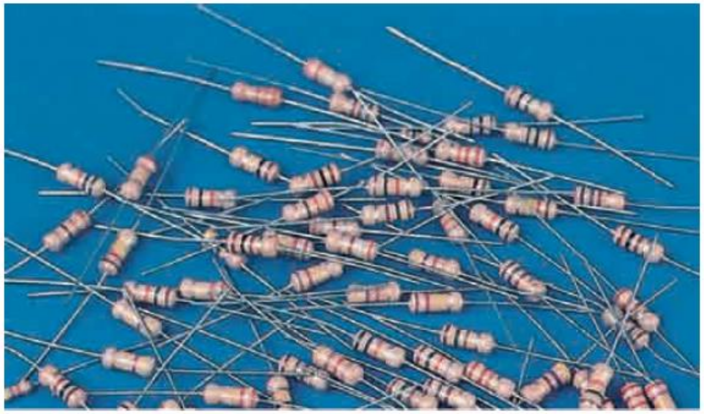
Resistance and Resistivity

We determine the resistance between any two points of a conductor by applying a potential difference *V* between those points and measuring the current *i* that results. The resistance *R* is then

$$R = \frac{V}{i} \qquad \text{(definition of } R\text{).}$$

The SI unit for resistance is the volt per ampere. This has a special name, the **ohm** (symbol Ω):

$$1 \text{ ohm} = 1\ \Omega = 1 \text{ volt per ampere} = 1 \text{ V/A}.$$



In a circuit diagram, resistors are represented by

***R* symbolized by** 

Resistance and Resistivity

The **resistivity**, ρ, of a resistor is defined as:

$$\rho = \frac{E}{J}$$

$$\rho = \frac{E}{J} = \frac{V/m}{A/m^2} = \frac{V.m}{A} = \Omega.m$$

The SI unit for ρ is $\Omega.m$.

The **conductivity** σ of a material is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho}$$

$$\vec{J} = \sigma\vec{E}.$$

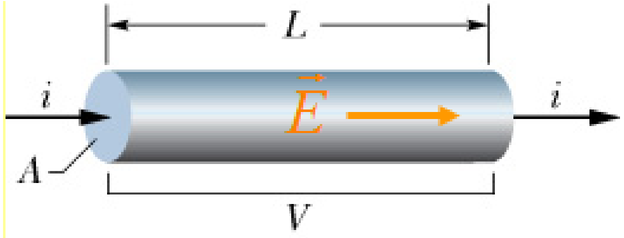
Resistance R is a property of an object.
Resistivity ρ is a property of a material

Table 26-1

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot$ m)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

Calculating Resistance from Resistivity



If the streamlines representing the current density are uniform throughout the wire, the electric field, *E*, and the current density, *J*, will be constant for all points within the wire.

$$E = V/L \quad \text{and} \quad J = i/A.$$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}.$$

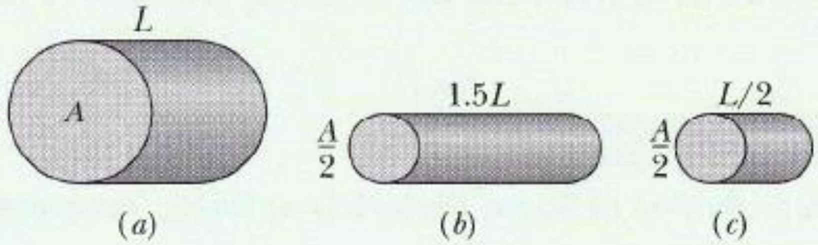
Where, $$R = \frac{V}{i}$$

Therefore,

$$\rho = R\frac{A}{L} \Rightarrow R = \rho\frac{L}{A}.$$

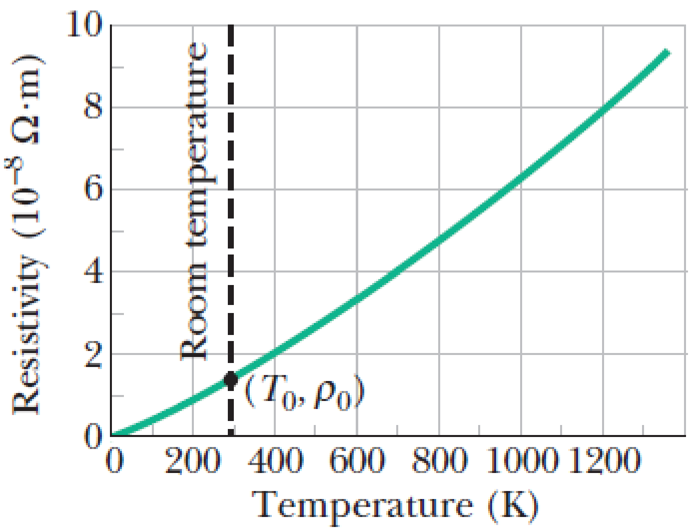
Calculating Resistance from Resistivity

CHECKPOINT 3 The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



Variation of R with temperature

Fig. 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$.



Resistivity can depend on temperature.

The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\rho - \rho_0 = \rho_0\, \alpha\, (T - T_0)$$

- ρ: Resistivity at T
- ρ_0: Resistivity at T_0
- α: Temperature coefficient of resistivity
- T: Temperature
- T_0: Reference temperature

Variation of R with temperature

A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions 1.2 cm × 1.2 cm) and (2) two rectangular sides (with dimensions 1.2 cm × 15 cm)?

Variation of R with temperature

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KEY IDEA

The resistance R of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio L/A, according to Eq. 26-16 ($R = \rho L/A$), where A is the area of the surfaces to which the potential difference is applied and L is the distance between those surfaces.

Calculations: For arrangement 1, we have $L = 15$ cm $= 0.15$ m and

$$A = (1.2\text{ cm})^2 = 1.44 \times 10^{-4}\text{ m}^2.$$

Substituting into Eq. 26-16 with the resistivity ρ from Table 26-1, we then find that for arrangement 1,

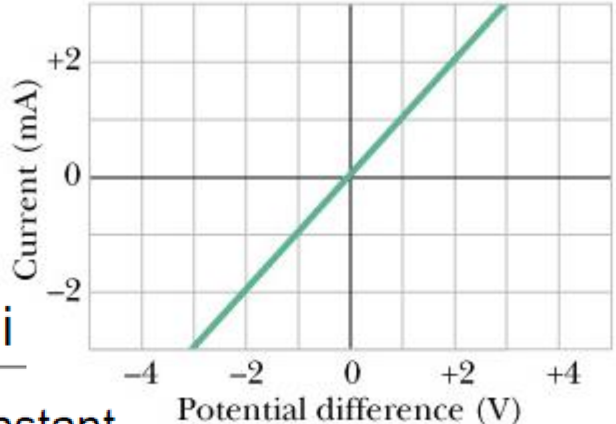
$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\,\Omega\cdot\text{m})(0.15\text{ m})}{1.44 \times 10^{-4}\text{ m}^2}3$$
$$= 1.0 \times 10^{-4}\,\Omega = 100\,\mu\Omega. \quad \text{(Answer)}$$

Similarly, for arrangement 2, with distance $L = 1.2$ cm and area $A = (1.2\text{ cm})(15\text{ cm})$, we obtain

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\,\Omega\cdot\text{m})(1.2 \times 10^{-2}\text{ m})}{1.80 \times 10^{-3}\text{ m}^2}$$
$$= 6.5 \times 10^{-7}\,\Omega = 0.65\,\mu\Omega. \quad \text{(Answer)}$$

Ohm's Law

Ohm's Law. A resistor was defined as a conductor whose resistance does not change with the voltage V applied across it.



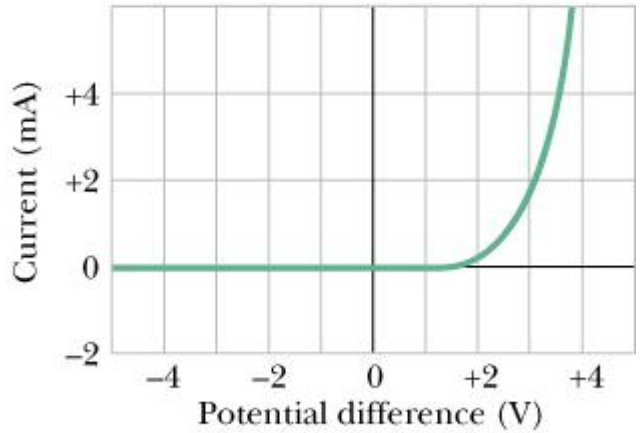
(*b*)

$V = R\,i$

Constant

Current is directly proportional to the potential difference

Device obeys Ohm's law



(*c*)

$V = R\,i$

Function of the potential difference

Device does not obey Ohm's law

Ohm's law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

$$i \propto V$$

Ohm's Law

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

$$R = \frac{V}{i}$$

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

$$\rho = \frac{E}{J}$$

Ohm's Law

CHECKPOINT 4 The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). From these data, determine which device does not obey Ohm's law.

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

Power in Electric Circuits

The amount of charge *dq* that moves between those terminals in time interval *dt* is equal to *i dt*.

This charge *dq* moves through a decrease in potential of magnitude *V*, and thus its electric potential energy decreases in magnitude by the amount

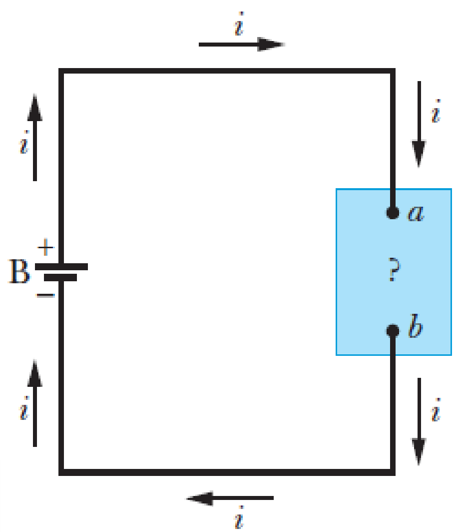
The power *P* associated with that transfer is the rate of transfer *dU*/*dt*, given by

$$P = iV \qquad \text{(rate of electrical energy transfer).}$$

$$P = i^2R \qquad \text{(resistive dissipation)}$$

$$\Rightarrow \quad P = \frac{V^2}{R} \qquad \text{(resistive dissipation).}$$

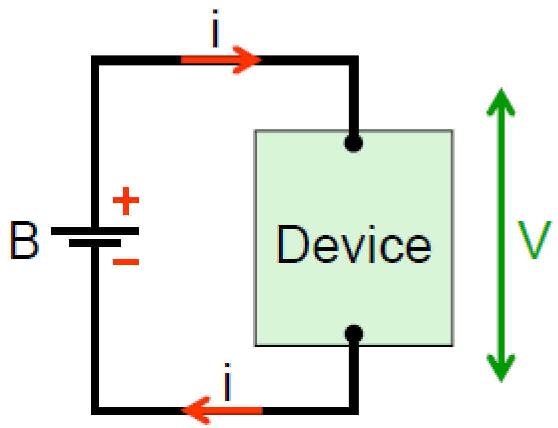
The battery at the left supplies energy to the conduction electrons that form the current.



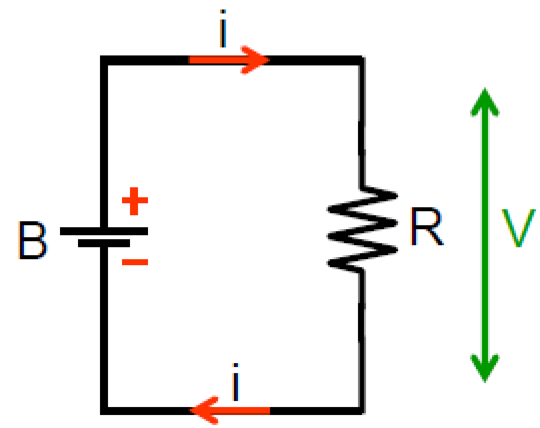
The unit of power is the volt-ampere (V A).

$$1\ \text{V}\cdot\text{A} = \left(1\ \frac{\text{J}}{\text{C}}\right)\left(1\ \frac{\text{C}}{\text{s}}\right) = 1\ \frac{\text{J}}{\text{s}} = 1\ \text{W}.$$

Power in Electric Circuits



Power is the rate of electric energy transfer from the battery to the device.



Power is the rate of electric energy transfer from the battery to the device.

$P = i\,V$

For any device
(resistor, motor, capacitor)

$P = i\,V$

For a resistor $V = R\,i$

$P = i^2\,R$

$$P = \frac{V^2}{R}$$

Power in Electric Circuits

CHECKPOINT 5 A potential difference V is connected across a device with resistance R, causing current i through the device. Rank the following variations according to the change in the rate at which electrical energy is converted to thermal energy due to the resistance, greatest change first: (a) V is doubled with R unchanged, (b) i is doubled with R unchanged, (c) R is doubled with V unchanged, (d) R is doubled with i unchanged.

Power in Electric Circuits

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of 72 Ω. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

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KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-26 to 26-28.

Calculations: Because we know the potential V and resistance R, we use Eq. 26-28, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{72\ \Omega} = 200\ \text{W}. \qquad \text{(Answer)}$$

In situation 2, the resistance of each half of the wire is (72 Ω)/2, or 36 Ω. Thus, the dissipation rate for each half is

$$P' = \frac{(120\ \text{V})^2}{36\ \Omega} = 400\ \text{W},$$

and that for the two halves is

$$P = 2P' = 800\ \text{W}. \qquad \text{(Answer)}$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

What have we learnt

- What is electric current (*i*)?
- What is current density (**J**) and drift speed (v_d)?
- What is resistance (R) and resistivity ($\rho = RA/L$)?
- Variation of R with temperature ($\rho - \rho_o = \rho_o \alpha (T - T_o)$)
- Ohm's law (V=i R) [Ohmic and non-Ohmic devices]
- How to find the power in an electric circuit?

($P = iV$, , $P = i^2R = V^2/R$)